LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2022**

PST 2501 – ESTIMATION THEORY

Date: 15-06-2022 Dept. No. Max. : 100 Marks

Time: 09:00 A.M. - 12:00 NOON

SECTION -A

Answer ALL the questions . **(10 x 2 = 20 Marks)**

1. Let $X_1,X_2,,,X_n$ be i.i.d. $N(0,\theta)$, $0 < \theta < \infty$. Obtain a sufficient statistic for θ.
2. Define Uniformly Minimum Risk Unbiased Estimator.
3. List the methods used to find UMVUE.
4. Define Fishers information.
5. Prove that $\{N(0,\sigma^2), \sigma^2 > 0\}$ is not complete.
6. State Lehmann-Scheffe theorem.
7. Show that Poisson distribution belongs to exponential family.
8. Let $X_1,X_2,...X_n$ denote a random sample of size n from $U[0,\theta]$, $\theta > 0$.Obtain M.L.E of θ.
9. Define CAN estimator and provide an example.
10. Write a note on Bootstrap method.

SECTION-B

Answer any FIVE questions **(5 x 8 = 40 Marks)**

11. Let $Y_1 < Y_2 < Y_3$ be the order statistics of a random sample of size 3 from the uniform distribution having pdf $f(x;\theta) = 1/\theta$, $0 < x < \infty$, $0 < \theta < \infty$, zero elsewhere.
Show that $4Y_1$, $2Y_2$ and $4/3\ Y_3$ are all unbiased estimators of θ. Further find the variance of each of these unbiased estimators.

12. Let $X_1,X_2,...X_n$ denote a random sample of size n from $N(\mu , \sigma^2)$, $\mu \in R$ and $\sigma^2 > 0$. Obtain sufficient statistic for (i) μ when σ^2 is known (ii) σ^2 when μ is known and (iii) (μ, σ^2) when μ *and* σ^2 are unknown. (3+2+3)

13. Let $X_1,X_2,...X_n$ be a random sample of size n from $b(1,\theta)$, $0 < \theta < 1$.
(a) Find Cramer-Rao Lower Bound for estimating θ. (6)
(b) Show that sample mean is a Minimum Variance Bound Estimator of θ. (2)

14. (a) State and prove Rao-Blackwell theorem. (4)
(b) Let $X_1,X_2,...X_n$ be a random sample with the common pdf $f(x;\theta) = \theta^{-1} \exp(-x / \theta)$ for $x > 0$, zero elsewhere. Determine the UMVUE of $1/\theta$. (4)

15. (a) Show with an example that MLE is not sufficient. (4)
(b) Let $X_1,X_2,...X_n$ be a random sample from $N(\theta,1)$, $\theta \in R$. Find a consistent estimator for θ. (4)

16. (a) Establish the invariance property of CAN estimator. (4)

(b) Find CAN estimator of $\exp(-\theta)$ in the case of $P(\theta)$, $\theta > 0$. (4)

17. Let $X \sim U(0,\theta)$, $\theta > 0$. Assume that the prior distribution of θ is $h(\theta) = \theta \exp(-\theta)$, $\theta > 0$.

Find Bayesian estimator of θ if loss function is

(a) squared error and (b) absolute error. (4+4)

18. (a) When is a distribution said to belong to exponential family ? (2)

(b) Show that $\{N(\theta,1), \theta \in R\}$ *is complete.* (6)

SECTION-C

Answer any TWO questions. **(2 x 20 = 40 Marks)**

19. (a) State and prove the necessary and sufficient condition for an estimator to be UMVUE using uncorrelated approach. (14)

(b) If δ_1^* is a UMVUE and δ_2^* is bounded UMVUE then show that $\delta_1^* . \delta_2^*$ is also UMVUE. (6)

20. Prove that MLE is not consistent.

21. (a) Let $X_1, X_2, ... X_n$ be a random sample from $N(\theta,1)$, $\theta \in R$. Show that sample mean and variance are independent using Basu's theorem. (12)

(b) Establish invariance property of MLE and illustrate with an example. (8)

22. (a) Let $X_1, X_2, ... X_n$ be a random sample from $b(1, \theta)$; $0 < \theta < 1$ and θ follows beta distribution of first kind with parameters α and β. Find Bayes' estimator of θ when loss is squared error. (10)

(b) Let Y have a binomial distribution in which n = 20 and $p = \theta$. The prior probabilities of θ are $P(\theta = 0.3) = 2/3$ and $P(\theta = 0.5) = 1/3$. If y = 9 , what are the posterior probabilities for $\theta = 0.3$ and $\theta = 0.5$? (6)

(c) Explain Jackknife method . (4)

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