LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2022**

PST 2502 – TESTING STATISTICAL HYPOTHESES

Date: 17-06-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

SECTION – A **(10 x 2 = 20)**

Answer ALL the Questions. Each carries 2 marks.

1. Distinguish between simple and composite hypotheses.
2. What is the difference between the level and size of the test?
3. Let β denote the power of a most powerful test of level α for testing simple hypothesis H_0 against simple alternative H_1. Prove that β ≥ α.
4. Define Multiparameter Exponential Family.
5. What are nuisance parameters and how do you deal with them?
6. Define Unbiased test.
7. When do we say that a test ϕ has Neyman Structure?
8. What is maximal invariant function?
9. Briefly explain the principles of LRT.
10. Give an example of a group with location changes.

SECTION – B **(5 x 8 = 40)**

Answer any FIVE Questions. Each carries 8 marks.

11. Let $X_1, X_2, \ldots, X_n$ be iid B(1 ,p) random variables. Find the Most powerful test function of level α for testing $H_0: p = p_0$ Vs $H_1: p = p_1$ $(p_1 > p_0)$.
12. (a) Define Monotone Likelihood Ratio.
    (b) Give an example for Non exponential family of distribution possessing MLR property.
13. Let $X_1, X_2, \ldots, X_n$ be a random sample from N(θ , 1). Show that UMP level α test does not exist for testing $H_0: \theta = \theta_0$ Vs $H_1: \theta \neq \theta_0$
14. Explain the need of bounded completeness to prove that similar tests have a Neyman structure?
15. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$ $\sigma > 0$ Derive UMPUT for $H_0: \sigma = \sigma_0$ $(\sigma^2 = \sigma_0^2)$Vs $H_1: \sigma \neq \sigma_0, (\sigma^2 \neq \sigma_0^2)$. Discuss the case when n=4, $\sigma_0 = 1, \alpha = 0$.
16. Explain Locally most powerful test. Establish the condition to obtain the LMPT.
17. Let $X_1, X_2, \ldots X_n$ be iid $N(\mu, \sigma^2)$, Find the shortest length confidence interval for μ with level 1-α based on a minimal sufficient statistic.
18. If X ~ P (λ_1) and Y ~ P (λ_2) and are independent, then compare the two Poisson populations through UMPUT for $H_0: \lambda_1 \leq \lambda_2$ versus $H_1: \lambda_1 > \lambda_2$, by taking random sample from P (λ_1) and P (λ_2) of sizes 'm' and 'n' respectively.

SECTION – C **(2 x 20 =40)**

Answer any TWO Questions. Each carries 20 marks.

19. State and prove the necessary and sufficient conditions of fundamental Neyman – Pearson lemma.

20. (a)Let $X_1, X_2, \ldots, X_n$ be the random sample of size n from $N(0, \sigma^2)$, $\sigma>0$. For testing $H_0: \sigma \leq \sigma_0$ Vs $H_1: \sigma > \sigma_0$. Derive UMPT with level α. Examine whether the test is consistent.

    (b) Consider the (k+1) parameter exponential family. Derive the conditional UMPU level α test for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$.

21. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(0, \theta)$ $\theta > 0$. Show that there exist a UMP level α test for $H_0: \theta = \theta_0$ Vs $H_1: \theta \neq \theta_0$.

    (b)Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu_1, \sigma^2)$ and $Y_1, Y_2, \ldots, Y_n$ be a random sample from $N(\mu_2, \sigma^2)$.Derive an unconditional UMPUT of size-α for testing $H_0: \mu_2 - \mu_1 \leq 0$ Vs $H_1: \mu_2 - \mu_1 > 0$

22. (a) Derive a likelihood ratio test for testing $H_0: \mu = \mu_0$ Vs $H_1: \mu \neq \mu_0$ when a random sample of size n is drawn from $N(\mu, \sigma^2)$.

23. (b)Let $X_1, X_2 \sim$ iid random sample $H_0: X_1, X_2$ are iid $N(\mu, 1)$ $\mu \in R$,

    $H_1: X_1, X_2$ are iid $C(\mu, 1)$, Cauchy with location μ. Find UMPIT w.r.t the group of translations.

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