LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

U.G. DEGREE EXAMINATION – **ALLIED**

FOURTH SEMESTER – **APRIL 2022**

UST 4401 – PROBABILTY AND DISTRIBUTIONS

Date: 27-06-2022 Dept. No. Max. : 100 Marks
Time: 09:00 AM - 12:00 NOON

Section – A

Answer ALL the questions **(10 X 2 = 20)**

1. Define joint probability density function.
2. Write the difference between discrete and continuous random variables.
3. State the properties of distribution function.
4. A random variable X is distributed at random between the values 0 and 1 so that its probability density function is $f(x) = kx^2(1-x^3)$, where k is a constant. Find the value of k.
5. State the conditions under which binomial distribution tends to Poisson distribution.
6. In a book of 520 pages, 390 typo-graphical errors occur. Assuming Poisson law for the number of errors per page, find the probability that a random sample of 5 pages will contain no error.
7. Write the mean and variance of beta distribution of second kind.
8. If X is uniformly distributed with mean 1 and variance 4/3, find P(X<0).
9. Define Markov chain.
10. State the difference between random variables and random processes.

Section – B

Answer any FIVE questions **(5 X 8 = 40)**

11. State and prove multiplication theorem of probability for n events.
12. A random variable X has the following probability function:

x	-2	-1	0	1	2	3
p(x)	0.1	k	0.2	2k	0.3	k

    (i) Find the value of k, and calculate mean and variance.
    (ii) Construct the c.d.f F(x).
13. Derive the m.g.f of Poisson distribution and hence find its mean and variance.
14. A coffee connoisseur claims that he can distinguish between a cup of instant coffee and a cup of percolator coffee 75% of the time. It is agreed that his claim will be accepted if he correctly identifies atleast 5 of the 6 cups. Find his chances of having the claim (i) accepted (ii) rejected, when he does have the ability he claims.
15. Prove that the exponential distribution has lack of memory property.
16. Define random processes and explain classification of processes.
17. The odds that a book on statistics will be favourably reviewed by 3 independent critics are 3 to 2, 4 to 3 and 2 to 3 respectively. What is the probability that of the three reviews: (i) All will be favourable, (ii) Majority of the reviews will be favourable, and (iii) Exactly two reviews will be favourable.

18. The joint probability distribution of two random variables X and Y is given by: P(X=0,Y=1) = 1/3, P(X=1,Y=-1) = 1/3, and P(X=1,Y=1) = 1/3. Find (i) Marginal distributions of X and Y and (ii) Conditional probability distribution of X given Y and Y given X.

Section – C

Answer any TWO questions **(2 X 20 = 40)**

19. (i) State and prove Bayes' theorem for n events. (12)

(ii) A factory produces a certain type of outputs by three types of machines. The respective daily production figures are: Machine I: 3000 units; Machine II: 2500 units; Machine III: 4500 units. Past experience shows that 1 percent of the output produced by Machine I is defective. The corresponding fraction of defectives for the other two machines is 1.2 percent and 2 percent respectively. An item is drawn at random from the day's production run and is found to be defective. What is probability that it comes from the output of (a) Machine I, (b) Machine II, (c) Machine III? (8)

20. (i) Derive the m.g.f of normal distribution and hence find its mean and variance. (10)

(ii) The local authorities in a certain city install 10000 electric lamps in the streets of the city. If these lamps have an average life of 1000 burning hours with a standard deviation of 200 hours, assuming normality, what number of lamps might be expected to fail (a) in the first 800 burning hours? (b) Between 800 and 1200 burning hours? (10)

21. (i) Derive the lack of memory property of geometric distribution. (12)

(ii)A person owning a scooter has the option to switch over to scooter, bike or car next time with the probability of (0.3 0.5 0.2). If the tpm is $\begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.2 & 0.5 & 0.3 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$, what are the probabilities of the vehicles related to his fourth purchase? (8)

22. (i) A manufacturer, who produces medicine bottles, finds that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Using Poisson distribution, find how many boxes will contain: (a) no defective and (b) atleast two defectives. (10)

(ii)Let $f(x,y) = 8xy,\ 0 < x < y < 1\ ;\ f(x,y) = 0,\ elsewhere$. Find (a) E(Y|X=x) and (b) Var(Y | X=x). (10)

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