LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIFTH SEMESTER – **APRIL 2022**

UST 5501 – APPLIED STOCHASTIC PROCESSES

Date: 15-06-2022 Dept. No. Max. : 100 Marks
Time: 09:00 AM - 12:00 NOON

PART – A

Answer ALL the Questions **(10X2=20)**

1. Write the postulates of Poisson process.
2. What is transition probability matrix?
3. Define stochastic process.
4. Stationary distribution and limiting distribution are same or not? Illustrate with example.
5. Define recurrent and transient states.
6. Consider the transition probability matrix $P = \begin{bmatrix} 0.33 & 0.67 \\ 0.75 & 0.25 \end{bmatrix}$, verify whether P is regular.
7. Explain extinction probabilities.
8. Define branching process.
9. Show that the regular chains are strict subclass of ergodic chain.
10. Define Markov chain.

PART – B

Answer any FIVE Questions **(5X8=40)**

11. Prove that a state i is recurrent if and only if $\sum_1^\infty P_{ii}^n = \infty$.
12. State and prove Chapman-Kolmogorov equation.
13. Find the classes and periodicity for the transition probability matrix

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1/3 & 0 & 2/3 & 0 \end{bmatrix}$$

14. Consider $P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}, 0 < a, b < 1$. Prove that $P^n = \frac{1}{a+b}\begin{bmatrix} b & a \\ b & a \end{bmatrix} + \frac{(1-a-b)^n}{a+b}\begin{bmatrix} a & -a \\ -b & b \end{bmatrix}$
15. Define the PGF of a Poisson distribution and hence find its mean and variance.
16. Derive Kolmogorov backward and forward differential equation.
17. Let P(s) be the PGF associated with offspring distribution $\{P_k\}$ and $P_n(s)$ that associated with X_n. Then show that $P_n(s) = P_{n-1}(P(s)) = P_1(P_{n-1}(s))$ provided $X_0 = 1$.
18. Explain Yule Furry process.

PART – C

Answer any TWO Questions **(2X20=40)**

19. Define Poisson process and derive the expression for $P_n(t)$.
20. i) Let P be the regular transition probability matrix on the state 0, 1,…,n. Then prove that the limiting distribution $\pi = (\pi_0, \pi_1, \dots, \pi_n)$ is the unique non negative solutions of $\pi_j = \sum_{k=0}^{N} \pi_k \, P_{kj}; j = 0, \dots, N$ and $\sum_{k=0}^{N} \pi_k = 1$.
    ii) Find the stationary probability distribution of the Markov chain with S = {1,2,3} whose transition probability matrix is
    $$\begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$ (10+10)
21. Derive pure birth process.
22. i)Find the variance of X_n using the properties of generating functions of branching process.
    ii) If an organism can either die or spilt into 2 with probabilities ½ each. What is the probability that second generation size is at most 2 , is exactly 4. (12+8)

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