LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2023**

PST2MC02 – TESTING STATISTICAL HYPOTHESIS

Date: 04-05-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION A – K1 (CO1)

	Answer ALL the questions	**(5 x 1 = 5)**
1.	**Answer the following**	
a)	Explain the method of obtaining the most powerful test function using the concept of linear programming problem.	
b)	Let X be a random variable with pdf $f(x)=\begin{cases}\theta e^{-\theta x} & 0<x<\infty, \theta>0 \\ 0 & otherwise\end{cases}$. Consider the problem of testing H_0: θ = 1 Vs H_1: θ = 2. Define $\phi(x)=\begin{cases}1 & if \quad x \geq 1 \\ 0 & if \quad x<1\end{cases}$. Find the level of the test.	
c)	When do we say a test function is consistent?	
d)	Explain Locally most powerful test.	
e)	What is the difference between parametric and non-parametric tests?	

SECTION A – K2 (CO1)

	Answer ALL the questions	**(5 x 1 = 5)**
2.	**Answer the following**	
a)	Distinguish between randomized and non-randomized test.	
b)	State the Generalized Neyman-Pearson Theorem.	
c)	Prove that UMPT is UMPUT.	
d)	Write any two assumptions of Non parametric Methods.	
e)	Explain the concept of likelihood ratio test in regression analysis.	

SECTION B – K3 (CO2)

	Answer any THREE of the following	**(3 x 10 = 30)**
3.	Let β denote the power of a most powerful test of level α for testing simple hypothesis H_0 against simple alternative H_1. Prove that (i) β ≥ α and (ii) α < β unless $p_0 = p_1$.	
4.	Let $X_1, X_2, \ldots, X_n$ be iid B(1 ,p) random variables. Find the Most powerful test function of level α for testing H_0: $p = p_0$ Vs $H_1: p = p_1$ $(p_1 > p_0)$.	
5.	Show that a test is invariant if and only if it is a function of a maximal invariant statistic.	
6.	Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$ $\sigma > 0$. Derive UMPUT for testing H_0: $\sigma = \sigma_0$ $(\sigma^2 = \sigma_0^2)$Vs $H_1: \sigma \neq \sigma_0, (\sigma^2 \neq \sigma_0^2)$. Discuss the case when n=4, $\sigma_0 = 1, \alpha = 0$.	
7.	Do we require bounded completeness to prove similar tests to have Neyman structure? Explain.	

SECTION C – K4 (CO3)

	Answer any TWO of the following **(2 x 12.5 = 25)**
8.	Derive a UMP test of level α for testing $H_0 : \theta \leq \theta_0$ Vs $H_1 : \theta > \theta_0$ for the family of densities $\{f(x,\theta), \theta \in \Theta\}$ that possess MLR in T(x). Also, show that the power function of the above testing problem increases in θ
9.	Consider the one parameter exponential family of distributions. Obtain the UMPT of level α for testing the two-sided testing hypothesis.
10.	Does UMPT exist always? Explain with an example.
11.	Derive a likelihood ratio test for testing $\mu = \mu_0$ Vs $H_1: \mu \neq \mu_0$ when a random sample of size n is drawn from N(μ, σ^2).

SECTION D – K5 (CO4)

	Answer any ONE of the following **(1 x 15 = 15)**
12.	a. Consider the (k+1) parameter exponential family. Derive the conditional UMPU level α test for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$. (10) b. Let $X_1, X_2, \ldots, X_n$ be the random sample of size n from N(0, σ^2), σ>0. For testing H_0: $\sigma \leq \sigma_0$ Vs H_1: $\sigma > \sigma_0$. Derive UMPT with level α. Examine whether the test is consistent. (5)
13.	If X ~ P (λ_1) and Y ~ P (λ_2) and are independent, then compare the two Poisson populations through UMPUT for H_0: $\lambda_1 \leq \lambda_2$ versus H_1: $\lambda_1 > \lambda_2$, by taking random sample from P (λ_1) and P (λ_2) of sizes 'm' and 'n' respectively.

SECTION E – K6 (CO5)

	Answer any ONE of the following **(1 x 20 = 20)**
14.	State and prove the Neyman – Pearson Fundamental lemma.
15.	a. Let $X_1, X_2, \ldots, X_n$ be iid N(μ,σ^2) random variables. Derive the unconditional UMPUT of level α for testing H_0: $\sigma^2 \leq \sigma_0^2$ Vs H_1: $\sigma^2 > \sigma_0^2$. (10) b. Explain the applications and procedure of chi -square test. (5) c. Obtain the test function for the testing the mixing proportion in the case of mixture distributions. (5)
