# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **NOVEMBER 2014**

**ST 3506 - MATRIX AND LINEAR ALGEBRA**

Date : 03/11/2014 Dept. No. Max. : 100 Marks

Time : 09:00-12:00

## SECTION A

**Answer ALL the questions.** **(10 X 2 = 20 Marks)**

1. Define a diagonal matrix with an example.

2. Show with an example that AB = 0 does not imply A = 0 or B = 0

3. Define minor and cofactor of an element in a matrix.

4. Find the value of $\begin{vmatrix} x^2 & 1 \\ 2 & x^2 \end{vmatrix}$

5. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ find $A^{-1}$.

6. Define linearly dependent vectors.

7. State any two properties of rank of matrices.

8. Define Basis of a vector space.

9. Define a linear transformation.

10. Show that if $\lambda$ is an eigen value of A, then $\lambda^2$ is an eigen value of $A^2$.

## SECTION B

**Answer any FIVE questions.** **(5 x 8 = 40 Marks)**

11. Prove that if A and B are symmetric matrices, then AB is symmetric if and only if AB = BA.

12. Show that if A and B commute, then every power of A commutes with every power of B.

13. Show that the set $V = \{ (x_1, x_2)' \mid x_1 + x_2 = 0, x_1, x_2 \in R\}$ is a vector space over R. Identify a basis for this space.

14. Prove that every vector in a vector space can be uniquely represented as a linear combination of the vectors in a basis of that space.

15. Show that the system of equations $\mathbf{A}\boldsymbol{x} = \boldsymbol{b}$ has a solution if and only if Rank (A) = Rank $(\mathbf{A} \vdots \boldsymbol{b})$.

[P.T.O]

16. Show that

$$\begin{vmatrix} 0 & a & b & c \\ a & 0 & c & b \\ b & c & 0 & a \\ c & b & a & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & c^2 & b^2 \\ 1 & c^2 & 0 & a^2 \\ 1 & b^2 & a^2 & 0 \end{vmatrix}$$

17. Find the characteristic roots and vectors of $\begin{bmatrix} 1 & 6 \\ 1 & 2 \end{bmatrix}$.

18. Show how the product of two matrices is related to the composition of Linear Transformations.

## SECTION C

**Answer any TWO questions** **(2 x 20 = 40 marks)**

19. (a) Prove that every square matrix is uniquely expressible as the sum of symmetric and a skew symmetric matrix.

(b) Show that the only type of square matrix that commutes with every other square matrix is the scalar matrix. (10 + 10)

20. (a) Show that A. adj(A) = |A| I. Hence show that $|\text{adj } A| = |A|^{n-1}$.

(b) Prove that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2\,(a+b+c)^3$. (10 + 10)

21. (a) Find the inverse of the matrix given below using method of partitioning:

$$\begin{bmatrix} 0 & 0 & 1 & 2 \\ 1 & -2 & 2 & 8 \\ 1 & -1 & 3 & 14 \\ 0 & 1 & 2 & 7 \end{bmatrix}$$

(b) Find the rank of the matrix given below:

$$\begin{bmatrix} 1 & 3 & 4 & 2 \\ 2 & 4 & 6 & 2 \\ -1 & 5 & 4 & 6 \end{bmatrix}$$

(15 + 5)

22. (a) State and prove Cayley-Hamilton Theorem

(b) Using Cayley-Hamilton Theorem, find the inverse of $\begin{bmatrix} 1 & 2 & 4 \\ -2 & 3 & 0 \\ 3 & -1 & 0 \end{bmatrix}$ (12 + 8)

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