LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – NOVEMBER 2016

16PST1MC01 / ST 1820 - ADVANCED DISTRIBUTION THEORY

Date: 02-11-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Part –A

Answer **ALL** the questions: **(10 X 2 = 20)**

1. State the properties of a distribution function.
2. Write the pdf of a Binomial truncated at 0.
3. Write the marginal distribution of X_1 and X_2 in the case of Bivariate Poisson distribution.
4. Write the necessary and sufficient condition for the independence of two random variables X_1 and X_2 in terms of pgf.
5. Write the conditional distribution of X_1 given $X_2 = x_2$ for a Trinomial distribution.
6. Define a power series distribution.
7. What is the $E\left[X_1 | X_2 = x_2\right]$ for a Bivariate normal distribution.
8. Let X denote the number of throws when a die is thrown till the face six is obtained. Find E(X).
9. Write the pdf of the r^{th} Order Statistics.
10. Verify whether the Quadratic form is positive definite
    $Q(x_1, x_2) = x_1^2 + 2x_2^2 + 3x_3^2 + 4x_1x_2 + 3x_1x_3 + x_2x_3$.

Part-B

Answer any **Five** questions: **(5 X 8 = 40)**

11. Given $F(x) = \begin{cases} 0, & x < -1 \\ \frac{x+2}{4}, & -1 \le x \le 1 \\ 1, & 1 \le x \end{cases}$, decompose the distribution into discrete and continuous. Find the mean and variance.
12. Let X_1 and X_2 be i.i.d geometric random variables. Obtain the pdf of X_1 given $X_1 + X_2 = n$
13. Obtain the recurrence relation satisfied by the power series distribution.
14. Let $X_1, X_2, ...X_n$ be a random sample from $f(x) = 1 \quad 0 < x < 1$. Obtain the pdf of range.
15. State and prove the additive property of Bivariate Poisson distribution.
16. Obtain the relationship satisfied by the mean, median and mode of Lognormal distribution.
17. Obtain the MGF of Inverse Gaussian distribution.
18. Let X_1 have Gamma distribution G(α , p_1) and another independent variable X_2 have Gamma distribution G(α , p_2) .Obtain the pdf of $\frac{X_1}{X_1 + X_2}$.

Part-C

Answer any **TWO** Questions: **(2X 20 = 40)**

19. a) State and prove Skitovitch theorem.

b) Let X_1, X_2, X_3 be independent normal random variables such that

$E(X_1)=1, E(X_2)=3$, $E(X_3)=2$ and $\text{var}(X_1)=1, \text{var}(X_2)=2, \text{var}(X_3)=3$

Examine the independence of (i) $2X_1 + X_3$ and $X_1 - X_2$, (ii) $X_1 + X_2 - 2X_3$ and $X_1 - X_2$.

c) Examine the independence of $\overline{X}$ and S^2 using Skitovitch theorem for a random sample from $N(\mu, \sigma^2)$ (10+5+5)

20. a) Let $X_1, X_2, ... X_n$ be a random sample from $f(x) = \alpha e^{-\alpha x}$.

Let $D_k = (n-k+1)[X_{(k)} - X_{(k-1)}]$ $k = 1,2,....n$ then show that D_k's are i.i.d with pdf f .

Hence show that $X_{(1)}$ and $\sum_{k=2}^{n}[X_{(k)} - X_{(1)}]$ are independent.

b) Let X be a non-negative absolutely continuous random variable then show that X satisfies lack of memory iff X is Exponential. (10+10)

21. a) Let (X_1, X_2) has Bivariate Binomial with parameters n, p_1, p_2 and p_{12}. Show that X_1 given $X_2 = x_2$ is equal in distribution to $U_1 + V_1$ where U_1 and V_1 are independent . Hence Obtain the Correlation Coefficient between X_1 and X_2.

b) Show that for a Bivariate Normal distribution X_1 and X_2 are independent iff $\rho = 0$ **(14+6)**

22. a) Derive the MGF of non-central chi-square distribution

b) Let X have Poisson distribution with parameter λ and λ itself is a random variable having Gamma distribution $G(\alpha, v)$. Obtain the marginal distribution of X. **(12 +8)**

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