LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – NOVEMBER 2016

M 202 – GENERAL MATHEMATICS - II

Date: 02-11-2016 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Answer any SIX questions

1. (a) Find the sum to infinity of the series $1 + \frac{2}{6} + \frac{2\,.\,5}{6\,.\,12} + \frac{2\,.\,5\,.\,8}{6\,.\,12\,.\,18} + \cdots \infty$.

   (b) Prove that $\frac{e^2-1}{e^2+1} = \frac{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\cdots}{1+\frac{1}{2!}+\frac{1}{4!}+\cdots}$.

   (c) Prove that $log\left(\frac{n+1}{n}\right) = 2\left[\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \cdots\right]$. **(7+5+5)**

2. (a) Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$.

   (b) Find the rank of the matrix $B = \begin{pmatrix} 1 & 2 & 5 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{pmatrix}$. **(12+5)**

3. (a) Solve $p^2 - 2py = 3y^2$ where $p = \frac{dy}{dx}$.

   (b) Show that $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$ is an exact equation and hence solve it. **(7+10)**

4. (a) Solve $y = (x-a)p - p^2$.

   (b) Solve $(D^2 - 4D + 3)y = e^{2x} + cos2x$. **(5+12)**

5. (a) Find the equation of the plane passing through the points $(2,5,-3), (-2,-3,5)$ and $(5,3,-3)$.

   (b) Prove that the lines $\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2}$; $\frac{x+3}{-4} = \frac{y+1}{7} = \frac{z-4}{1}$ are coplanar. Find also their point of intersection and the plane through them. **(7+10)**

6. (a) Find the shortest distance between the lines $\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$; $\frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$.

   (b) Find the equation of the sphere through the four points $(2,3,1), (5,-1,2), (4,3,-1)$ and $(2,5,3)$. **(7+10)**

7. (a) Show that the vector $\vec{F} = 3y^4z^2\vec{i} + 4x^3z^2\vec{j} + 3x^2y^2\vec{k}$ is solenoidal.

   (b) Verify the divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by $x=0, x=1, y=0, y=1, z=0$ and $z=1$. **(5+12)**

8. (a) Form the partial differential equation by eliminating the arbitrary constants in $z = (x-a)^2 + (y-b)^2 + 1$.

   (b) Find the complete solution and singular integral of $z = px + qy + p^2q^2$.

   (c) Solve $z(x-y) = x^2p - y^2q$. **(5+5+7)**

9. (a) Find the Laplace transform of (i) $sinh6t + 3e^{-5t} + cos5t$ (ii) $(t+1)^2$

   (b) Solve the equation $y'' + 4y' - 5y = 5$ given that $y(0) = 0, y'(0) = 2$ using Laplace transform **(7+10)**

10. (a) Evaluate (i) $\int_0^1 x^7(1-x)^8dx$ (ii) $\int_0^{\frac{\pi}{2}} sin^7\theta\, cos^5\theta\, d\theta$.

    (b) Find the Fourier series of the function $f(x) = \begin{Bmatrix} -1, -\pi \le x < 0 \\ 1, 0 \le x \le \pi \end{Bmatrix}$. **(7+10)**

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