# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – NOVEMBER 2016

**ST 2814 - ESTIMATION THEORY**

Date: 03-11-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

## PART – A

**Answer ALL the questions** **(10 x 2 = 20)**

1. Explain the problem of Point estimation.
2. Give an example to prove that an unbiased estimator need not be unique.
3. Suggest an unbiased estimatorofθ, when arandom sample$X_1$, $X_2$,…,$X_n$drawn from U(0, θ).
4. If δ is a UMVUE, then show that δ + 2 is also a UMVUE.
5. Find which oneofthe followingis ancillarywhena random sample$X_1$, $X_2$is drawn from N(μ,1).
   (a)$X_1/X_2$ (b) $X_1+X_2$ (c)$X_1$-X2 (d) $2X_1-X_2$
6. State Neyman - Fisher Factorization Theorem
7. Give an example of a family of distributions which is not complete.
8. Define completeness andbounded completeness.
9. Suggest an MLEforP[X=0]in the caseofPoisson (θ).
10. Explain Jackknife method.

## PART– B

**Answer anyFIVE questions** **(5X8=40 marks)**

11. Let X be a discrete r.v. with $P(x;\theta) = \begin{cases} \theta & , x=-1 \\ (1-\theta)^2\,\theta^x & , x=0,1,2,... \end{cases}$
    Find all the unbiased estimators of 0.
12. Let X~ N (θ,1). Obtain the Cramer- Rao lower bound for estimating $\theta^2$. Compare the variance of the UMVUE with CRLB.
13. Let $X_1$, $X_2$,…,$X_n$ be a random sample of size n from $N(\mu,\sigma^2)$ , $\mu \in \mathbf{R}, \sigma > 0$ .
    Find Cramer-Rao Lower bound for estimating $\mu/\sigma$ . Compare it with the variance of UMVUE.
14. If T is sufficient for **P** or θ, then show that one-one function of T is also sufficient for **P** or θ. Illustrate with an example.
15. State and establish Lehmann-Scheffe theorem.
16. MLE is not unique – Illustrate with an example.
17. i. State Cramer-Rao regularity conditions
    ii. State and prove CR inequality.
18. Given a random sample from B(1,θ ), 0<θ<1, assuming that the priordistribution of θ to be a beta distribution, find a Baye's estimator of θ with respect to Squared Error loss function.

## PART– C

**Answer anyTWO questions** **(2 X 20 =40marks)**

19. (a) If UMVUE exists for the parametric function $\psi(\theta)$ then show that it must be essentially unique.

(b)Let $X_1,X_2$ be a random sample from E(0,σ). Show that $(X_1+X_2)$ and $X_1|(X_1+X_2)$ are independent using Basu's theorem. **(10+10)**

20. (a) Given an example of an MLE which is not CAN.

(b) Let $X_1,X_2,...,X_n$ be a random sample from population having pdf

$$p(x\mid\theta)=\begin{cases}\dfrac{1}{\theta}e^{-\frac{x}{\theta}}\ ,\ 0<x<\infty\ ,\ \theta>0\\ \\ 0, \qquad otherwise\end{cases}$$

Obtain MLE of P(X>2). **(10+10)**

21. (a) Let $(X_i,Y_i)$ , i=1,2,…,n be a random sample from ACBVE distribution with

pdf $f(x,y)=\{(2\alpha+\beta)(\alpha+\beta)/2\}\exp\{-\alpha(x+y)-\beta\max(x,y)\}, \qquad x,y>0.$

Find MLE of α and β.

(b) MLE is not consistent – Support the statement with an example. **(10+10)**

22. (a) "Blind use of Jackknifed method" – Illustrate with an example.

(b) Explain Baye's estimation with an example. **(10+10)**

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