LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – NOVEMBER 2016

ST 2815 - TESTING STATISTICAL HYPOTHESIS

Date: 05-11-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

SECTION – A : ANSWER ALL THE QUESTIONS (10 X 2 = 20)

1. Define power function of a test.
2. Define uniformly most powerful test.
3. State Generalized Neyman-Pearson Theorem
4. Show that an UMP test is unbiased
5. Define one parameter exponential family.
6. Define α-similar test
7. Mention any two properties of multi parameter exponential family.
8. Give an example of an invariant decision problem
9. Define maximal invariant?
10. Briefly explain the principles of LRT

SECTION – B: ANSWER ANY FIVE THE QUESTIONS (5X 8 = 40)

11 Let $X_1, X_2 \sim B(1,\theta)$. Let H: $\theta = 0.1, 0.2$ against K: $\theta = 0.3, 0.4$.

$$\text{Let the } \phi(x_1, x_2) = \begin{cases} 1 & \text{if } x_1 + x_2 = 0 \\ 0.1 & \text{if } x_1 + x_2 = 1 \\ 0 & \text{if } x_1 + x_2 = 2 \end{cases}$$

Find the size and Power of the given test function.

12 Let X be random variable with probability mass function under H and K are given by

X	1	2	3	4	5	6
$f_0(x)$	0.01	0.01	0.01	0.01	0.01	0.95
$f_1(x)$	0.05	0.04	0.03	0.02	0.01	0.85

Suppose α = 0.03, find the test function by using Neyman-Pearson's lemma and find the probability of Type II error and Power of the test.

13 Let $X_1, X_2, \ldots, X_n$ be a random sample from U(0, θ). Derive the UMP level test for testing H: $\theta \leq \theta_0$ against K: $\theta > \theta_0$. Also determine the cutoff point.

14 Let $X_1, X_2, \ldots, X_n$ be a random sample from a Cauchy distribution with parameter (1, θ). Show that this family does not have MLR property.

15 Let $X_1, X_2, \ldots, X_n$ be a random sample from p(λ). Show that it has Monotone Likelihood Ratio property in $\sum x_i$.

16 Let X have the distribution $P \in \mathcal{P}$ and T be a sufficient statistic for $\mathcal{P}$. Show that a necessary and sufficient condition for all similar tests have Neyman structure is that the family $\mathcal{P}^T$ of distributions of T is boundedly complete

17 Let $X_1, X_2, \ldots, X_n$ be a random sample from N(μ, σ^2), where σ^2 is known. Derive UMPT of level α for testing H: $\mu \leq \mu_0$ versus K: $\mu > \mu_0$

18 Obtain the Likelihood ratio Test for equality of means of 'k' Normal populations with a common variance.

SECTION – C: ANSWER ANY TWO QUESTIONS (2X 20= 40)

19 State and prove the necessary and sufficient condition of Neyman – Pearson fundamental Lemma. **(20)**

20 a) Two independently identically distributes random observations say X and Y are made on random variables whose distribution under the hypothesis H and K are given below. **(10)**

Values of X	0	1	2	3	4
P[X=x / H]	0.25	0.25	0.25	0.25	0.00
P[X=x / K]	0.1	0.2	0.3	0.3	0.1

Consider the test function $\phi(x, y) = \begin{cases} 1 \text{ if } x + y > 5 \\ 0.3 \text{ if } x + y = 5 \\ 0 \text{ if } x + y < 5 \end{cases}$

Find the size and power of the test function.

b) Let $X_1, X_2, \ldots X_n$ be a random sample of size n drawn from

$$f(x,\theta) = e^{-(x-\theta)};\ \theta < x < \infty .$$ **(10)**

Does $\{f_\theta(x)\}$ belong to the exponential family? Does $\{f_\theta(x)\}$ have MLR?

21 a) Let $X_1, X_2, \ldots X_m$ be a random sample from P(λ) and $Y_1, Y_2, \ldots Y_n$ be a random sample from P(μ). Derive UMPU level α test for testing the hypothesis H: $\lambda \le \mu$ against K: $\lambda > \mu$. **(10)**

b) Define multi Parameter exponential family. Also mention its objectives and properties. **(10)**

22 Let $X_1, X_2, \ldots X_m$ be a random sample from N(μ, σ_1^2) and let Let $Y_1, Y_2, \ldots Y_n$ be a random sample from N(δ, σ_2^2). Derive an unconditional UMPUT of level α for testing H: $\frac{\sigma_2^2}{\sigma_1^2} \le \Delta_0$ versus K: $\frac{\sigma_2^2}{\sigma_1^2} > \Delta_0$. **(20)**

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