LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **NOVEMBER 2016**

ST 3503/ST 3501/ST 3500 – STATISTICAL MATHEMATICS - II

Date: 04-11-2016 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A

Answer ALL questions: **(10 x 2 = 20 marks)**

1. Define Riemann integral.
2. Define the lower integral of a function.
3. Define improper integrals.
4. Define a gamma integral.
5. What is meant by change of order of integration?
6. Define variance – covariance matrix.
7. Define Poisson process.
8. Find the order and degree of the differential equation $\frac{d^2y}{dx^2}$ - 4 $\sqrt{\frac{dy}{dx}} = 5$.
9. State Cayley Hamilton theorem.
10. Define Eigen roots.

PART – B

Answer any FIVE questions: **(5 x 8 = 40 marks)**

11. Prove that if $f \in \mathcal{R}[a,b]$ and λ is any real number then λ $f \in \mathcal{R}[a,b]$ and $\int_a^b \lambda f = \lambda \int_a^b f$.
12. Prove that the improper integral $\int_1^{\infty} \frac{1}{x} dx$ diverges.
13. A continuous random variable X has a pdf given by f(x) = $\begin{cases} kxe^{-\lambda x}, x \geq 0, \lambda \geq 0 \\ 0; \quad Otherwise \end{cases}$. Determine the constant k. Obtain the mean and variance of X.
14. If y = x(x-3)(x-5), then find $\frac{dy}{dx}$.
15. The joint pdf of X and Y is f(x,y) = $e^{-(x+y)}$ x ≥ 0, y ≥ 0. Find the pdf of $\frac{X+Y}{2}$.
16. If $y = log\{x + \sqrt{a^2 + x^2}\}$, then find $\frac{dy}{dx}$.
17. Evaluate $\int_0^1 \frac{xdx}{x+\sqrt{1+x^2}}$.
18. Find the characteristic roots of $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$.

PART – C

Answer any TWO questions: **(2 x 20 = 40 marks)**

19. a) State and prove the fundamental theorem on calculus. (10 Marks)

    b) Prove that f ε $\mathcal{R}$[a,b] and a < c < b, then f ε $\mathcal{R}$[a,c], * f ε $\mathcal{R}$[c,b], and $\int_a^b f = \int_a^c f + \int_c^b f$. (10 Marks)

20. a) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (10 Marks)

    b). Use Laplace transform to solve the initial value problem $y'' + 3y' + 2y = 6e^{-t}$, y(0) = 1, $y'(0) = 2$. (10 Marks)

21. (X,Y) is a two – dimensional random variable with density function

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y), & 0 < x < 1, 0 < y < 1 \\ 0; & Otherwise \end{cases}$$

Find the conditional mean and conditional variance given $y = \frac{1}{2}$.

22. a) Solve the system of equations: $5X + 3Y + 14Z = 4$, $Y + 2Z = 1$, $X - Y + 2Z = 0$. (10 Marks)

b) Find the inverse of the matrix using Cayley's Hamilton theorem $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. (10 Marks)

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