LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – NOVEMBER 2016

ST 3816 - STOCHASTIC PROCESS

Date: 03-11-2016 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Section – A

Answer all the questions **10 x 2 = 20 marks**

1. Define a stochastic process with stationary independent increments.
2. Define a stochastic transition probability matrix.
3. Write any two properties of the period of a state of a Markov chain.
4. If $i \leftrightarrow j$ and if i is recurrent then show that j is recurrent.
5. Write the infinitesimal generator of birth and death process.
6. Define a renewal process.
7. State the renewal function.
8. Define a supermartingale.
9. Write the mean and variance of a discrete time branching process.
10. State any two examples for a stationary process.

Section – B

Answer any five questions **5 x 8 = 40 marks**

11. Explain the following:
    (a) Spatially homogeneous Markov chains
    (b) One-dimensional random walks
12. Define communication of states and show that communication is an equivalence relation.
13. Investigate the existence of a stationary probability distribution for the class of one dimensional random walks.
14. If a Markov chain on states {0,1,2,3,4,5} has the transition probability matrix

$$\begin{Vmatrix} 1/3 & 2/3 & 0 & 0 & 0 & 0 \\ 2/3 & 1/3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/4 & 3/4 & 0 & 0 \\ 0 & 0 & 1/5 & 4/5 & 0 & 0 \\ 1/4 & 0 & 1/4 & 0 & 1/4 & 1/4 \\ 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \end{Vmatrix}$$

find the classes and check for the recurrence of states.

15. Derive a system of differential equations for a pure birth process.
16. Explain the replacement models found in renewal theory.
17. Explain Martingale concept by providing two examples.
18. Establish any two properties of mean square distance.

Section – C

Answer any two questions **2 x 20 = 40 marks**

19. (a) If a process $\{X_t, t \in T\}$, where T= [0, ∞) or T = (0, 1,2,…) has stationary independent increments and has a finite mean, then show that $E[X_t] = m_0 + m_1 t$, where $m_0 = E[X_0]$ and $m_1 = E[X_1] - m_0$.

(b)Show that the three dimensional random walk is transient.

(5 +15) marks.

20. (a) State and prove the basic limit theorem of Markov chains.

(b) State and prove the stationary probability distribution of the Markov chain.

(10 + 10) marks.

21. (a)Derive $P_n(t)$ for a Poisson process clearly stating the assumptions.

(b) Derive the forward and backward Kolmogorov differential equations for the birth and death process.

(10 + 10) marks

22.(a) Explain branching process with an example.

(b) Establish the relationship of probability generating functions for branching process and hence find mean and variance.

(5 + 15)marks

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