LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FOURTH SEMESTER – **NOVEMBER 2016**

ST 4503/ST 5504/ST 5500 – ESTIMATION THEORY

Date: 17-11-2016 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART A

Answer ALL the questions. **(10 x 2 = 20)**

1. Suggest a sufficient estimator for θ based on a random sample from Poisson (θ).
2. Define statistic.
3. State factorization theorem.
4. Define completeness.
5. Mention any two methods of estimation.
6. Define UMVUE.
7. Define prior distribution.
8. Define confidence interval.
9. Is the unbiased estimator unique? Explain.
10. State the invariance property of MLE.

PART B

Answer any FIVE questions. **(5 x 8 = 40)**

11. State and prove Cramer – Rao inequality.
12. A random sample (X_1, X_2, X_3, X_4, X_5) of size 5 is drawn from a normal population with unknown mean μ. Consider the following estimators to estimate μ.

    (i) $t_1 = \frac{X_1+X_2+X_3+X_4+X_5}{5}$ (ii) $t_2 = \frac{X_1+X_2}{2} + X_3$ (iii) $t_3 = \frac{2X_1+X_2+\lambda X_3}{3}$

    where λ is such that t_3 is an unbiased estimator of μ.

    a) Find λ .
    b) Are t_1 and t_2 unbiased?
    c) Which estimator is the best among t_1, t_2 and t_3?

13. Obtain the MLE of θ in $f(x, \theta) = (1 + \theta) x^{\theta}$, $0 < x < 1$, based on an independent sample of size n.
14. State and prove Lehmann – Scheffe theorem.
15. Obtain the Bayes estimator of θ from Poisson distribution using a suitable prior distribution.
16. Obtain the MVB estimator for μ in normal population $N(\mu, \sigma^2)$, where σ^2 is known.
17. What are the regularity conditions for Cramer – Rao Inequality? Give an example where the condition is not satisfied.

18. Establish the uniqueness of UMVUE.

PART C

Answer any TWO questions. **(2 x 20 = 40)**

19. a) If T is a consistent estimator for θ and if g is a continuous function, then show that g(T) is consistent for g(θ).

    b) State and Prove the sufficient conditions for consistency of an estimator.

20. a) State and Prove Rao – Blackwell theorem.

    b) Obtain the UMVUE of θ, when a random sample of size n is drawn from b(1, θ)

21. a) Discuss the properties of MLE?

    b) Obtain the MLE of the parameters in N(μ,σ^2), when both the parameters are unknown.

22. a) Obtain 100(1 – α) % confidence interval for the parameter μ of the normal distribution when σ is unknown.

    b) Obtain the confidence interval for the proportion based on a random sample of size n.

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