LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIFTH SEMESTER – NOVEMBER 2016

ST 5400 - APPLIED STOCHASTIC PROCESSES

Date: 09-11-2016 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Part – A

Answer **ALL** the questions: **(10 x 2 = 20)**

1. Define a stochastic process with an example.
2. Define state space and time space of a stochastic process.
3. Define a Markov chain.
4. For the following Markov chain with states 0 and 1 and the transition probability matrix P = $\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$, which state is recurrent?
5. Obtain the periodicity of the States 0 and 1 in the Markov chain with transition probability matrix $P=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
6. Define communication of states i and j in a Markov chain.
7. What is the distribution of time between arrivals in a Poisson process? Write the pdf.
8. Let X(t) have a Poisson process with $\lambda = 2$. Obtain P[X(5) = 0].
9. State Basic limit theorem.
10. In an irreducible aperoidic, recurrent Markov Chain $\lim_{n \to \infty} p_{ij}^{n} =$ --------------

Part – B

Answer any **FIVE** questions: **(5 X 8 = 40)**

11. State and prove Chapman – Kolmogorov equation for an n – step transition probability matrix
12. Determine the Classes and periodicity of the states 0,1,2,3 of the Markov chain with the following transition probability matrix

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 \end{bmatrix}$$

13. For the Markov chain with states 0,1,2 and the transition probability matrix

$$P=\begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$$

and the initial distribution $P[X_0 = i] = \frac{1}{3}$, i = 0,1,2

Obtain i) $P[X_2 = 2, X_1 = 1 | X_0 = 2]$

ii) $[X_2 = 2, X_1 = 1 | X_0 = 1]$

iii) $[X_2 = 2 | X_0 = 1]$ **(2+2+4)**

14. Explain the classifications of a stochastic process with examples.

15. State and prove the additive property of Poisson process.

16. Verify whether the following Markov chain with states 0,1,2, is irreducible and aperiodic

$$P=\begin{pmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

17. Explain pure birth process.

18. Explain Poisson Process with examples.

Part- C

Answer any two questions **2 x 20 = 40**

19. Obatin the stationary distribution π_i, i=0,1,2 for the Markov chain with transition

probability matrix $P=\begin{pmatrix} 1/3 & 0 & 2/3 \\ 0 & 2/3 & 1/3 \\ 2/3 & 1/3 & 0 \end{pmatrix}$

20. Obtain the expression $p_n(t)$ for a Poisson process after stating the postulates.

21. i) Show that if j is recurrent in a Markov chain then $\sum_n P_{jj}^n = \infty$

ii) If $i \leftrightarrow j$, Show that if i is recurrent then j is also recurrent.

22. Write short notes on any three of the following: a) Discrete queuing Markov chain,

b) Random walk, c) Stationary independent increments and d) Can all the States be transient in a finite Markov chain?

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