LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIFTH SEMESTER – NOVEMBER 2016

ST 5503 / ST 5507 - COMPUTATIONAL STATISTICS

Date: 07-11-2016 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

Answer any three questions. Each question carries 34 marks.

1. (a) In a population of size N = 5 , values of the population characteristic are 1,3,5,7 and 9. A sample of size n=2 is drawn without replacement. Verify that $\bar{y}$ is an unbiased estimate of $\bar{Y}$ and that Var($\bar{y}$) is(S^2/n). (N-n) /N . **(6 marks)**

   (b) A population of size 100 is divided into four strata whose sizes are $N_1 = 40$,$N_2 = 10$,$N_3 = 20$ and $N_4 = 30$ and variances (S_i^2) are 2,3,4,5 respectively. How will you draw samples of size 20 according to the two allocations(proportional and optimum) ? **(6 marks)**

   (c) The following random sample (without replacement) is drawn from a population of 100 units:

   5, 6, 3, 3, 2, 3, 3, 3, 4, 4

   Find (i) the sample mean (ii) the sample variance (iii) an unbiased estimate of the population variance (iv) an unbiased estimate of variance of sample mean (v) an estimate of the standard error of mean. **(10 marks)**

   (d) 2000 cultivator's holdings in U.P. are stratified according to size. The number of holdings(N_h), mean area under wheat per holding ($\bar{Y}_h$) and standard deviation of area under wheat per holding(S_h) are given below for each stratum.

Stratum no.	Holding size (acres)	No. of holdings(N_h)	$\bar{Y}_h$	S_h
1	0 - 40	394	5.4	8.3
2	41 - 80	461	16.3	13.3
3	81 -120	391	24.3	15.1
4	121-160	334	34.5	19.8
5	161-200	169	42.1	24.5
6	200and above	261	57.9	31.2

A sample of 100 holdings is taken to estimate the mean area under wheat per holding by

(i) Simple random sampling (ii) stratified random sampling with proportional allocations

(iii) stratified random sampling with optimum allocations.

Compare the standard errors of the estimates in the three cases. **(12 marks)**

2. (a) Find the maximum likelihood estimates for $\theta_1 = \mu$ and $\theta_2 = \sigma^2$ if a random sample of size 15 from N(μ , σ^2) yielded the following values: 31.5 36.9 33.8 30.1 33.9 35.2 29.6 34.4 30.5 34.2 31.6 36.7 35.8 34.5 32.7. **(6 marks)**

   (b) As a clue to the amount of organic waste in Lake Macatawa, a count was made of the number of bacteria colonies in 100 milliliters of water. The number of colonies, in hundreds, for n = 30 samples of water from the east basin yielded the following observations:

   93 140 8 120 3 120 33 70 91 61 7 100 19 98 110 23 14 94 57 9 66 53 28 76 58 9 73 49 37 92.

   Find an approximate 95% confidence interval for the mean number of colonies in 100 milliliters of water in the east basin, μ_E. **(8 marks)**

   (c) A botanist measured the growths of pea stem segments, in millimeters , for n = 11 observations of

   X : 0.8 1.8 1.0 0.1 0.9 1.7 1.0 1.4 0.9 1.2 0.5 and m=13 observations of

   Y: 1.0 0.8 1.6 2.6 1.3 1.1 2.4 1.8 2.5 1.4 1.9 2.0 1.2.

   Test the hypothesis $H_0 : \sigma^2_X = \sigma^2_Y$ against $H_1 : \sigma^2_X \neq \sigma^2_Y$ at $\alpha = 0.05$. **(12 marks)**

(d) The intelligent quotient of 10 students are as follows:
110 120 145 160 128 126 138 155 129 140 .
Test $H_0 : \mu = 148$ against $H_1 : \mu \neq 148$ at $\alpha = 0.01$. **(8 marks)**

3. (a) The number of mistakes per page in a book of 259 pages are given below:

No.of mistakes per page (x) :	0	1	2	3	4	5
No. of pages(f) :	158	60	22	12	5	2

Fit a Poisson distribution to the given data and test the goodness of fit at 5% level of significance. **(15 marks)**

(b) The random samples from two different populations are given below:

X :	54	65	58	90	43	56	76	86	50	70
Y :	60	70	50	55	85	35	60	72	49	54

Test $H_0 : \mu_X = \mu_Y$ against $H_1 : \mu_X \neq \mu_Y$ at $\alpha = 0.05$. **(10 marks)**

(c) Let X and Y equal the percentages of body fat for women and men , respectively with distribution functions F(x) and G(y).Using runs test the hypothesis H_0: F(z) = G(z) against H_1: F(z) < G(z) at $\alpha = 0.05$. Ten observations of both X and Y that have been ordered are

X :	16.6	16.7	18.5	19.2	21.5	22.4	22.6	23.2	24.2	26.3
Y:	9.4	9.7	11.3	11.8	13.3	15.6	16.1	16.5	18.2	21.7

(9 marks)

4.(a) For the following data compute Fisher , Dorbish-Bowley , Marshall-Egeworth and Walsh price and quantity index numbers:

Commodity	Base year price	Current year price	Base year quantity	Current year quantity
A	40	50	10	7
B	20	30	5	8
C	30	40	6	10
D	10	20	9	10

Also verify the time and factor reversal tests. **(16 marks)**

(b) A shoe maker in the city A earns Rs.450 per month. The cost of living index number for a particular month is given as Rs.140. Using the following data find out the amount he spends on food and clothing:

Group	: Food	clothing	House rent	Fuel & lighting	Miscellaneous
Expenditure(in Rs)	: ?	?	100	60	90
Group index	: 150	120	150	115	140

(8 marks)

(c) Given below are two sets of indices:

Year	: 1939	1940	1945	1947	1949	1950	1951	1952
Index (old) A :	100	110	115	125	150	…	...	...
Index (new) B :	…	…	…	…	100	105	120	130

(i) Splice new series to old series. (ii) Splice old series to new series. **(6 marks)**

(d)The annual wages of a worker in rupees along with price index numbers are given below:

Year	: 1971	1972	1973	1974	1975
Wages	: 220	225	250	278	315
Index number	: 100	110	125	135	150

Prepare index numbers for real wages of workers. **(4 marks)**

5.(a) Fit a straight line trend by least squares to the following data and calculate trend values:

Year	: 1985	1986	1987	1988	1989	1990
Production ('000 tons) :	75	83	109	129	134	148

(14 marks)

(b) Find out the seasonal indices by the method of moving averages for the following data:

Quarter / Year	I	II	III	IV
2001	30	40	36	34
2002	34	52	50	44
2003	40	58	54	48
2004	54	76	58	62
2005	80	92	80	82

(20 marks)