LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **NOVEMBER 2017**

16UST3MC02 – ESTIMATING THEORY

Date: 07-11-2017 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

PART A

Answer **ALL** the questions: **(10X2=20 marks)**

1. Define unbiased estimator.
2. State the use of Lehmann-Scheffe Theorem in Estimation Theory.
3. Define sufficient statistic.
4. State any two properties of UMVUE.
5. Suggest an Moment estimator of Poisson Distribution with the parameter ª .
6. Describe method of least square estimation.
7. Define loss function.
8. Define prior distribution.
9. What are confidence intervals?
10. Let $X_1, X_2, ..., X_n$ be a random sample of size n from N(μ,σ^2), σ > 0 and μ is known. Write

    100 (1 − ~)% confidence interval for ± ^{2} .

PART B

Answer any **FIVE** questions: **(5X8=40 marks)**

11. State and prove a sufficient condition for an estimator to be consistent.
12. Derive the Cramer-Rao Lower Bound for estimating μ in N(μ, 1), and obtain minimum variance bound unbiased estimator for « .
13. Let $X_1, X_2, ..., X_n$ be a random sample from Poisson distribution P(λ), λ > 0. Show that $\sum_{i=1}^{n} X_i$ is a complete sufficient statistic.
14. State and prove Lehmann-Scheffe theorem.
15. For B(1,θ), show that $\overline{X}$ is Minimax Estimator of θ in the class $\mathscr{T}$ = { $\overline{X}$ + α : α ∈ ℝ} with respect to squared-error loss .
16. Let $X_1, X_2, ..., X_n$ be a random sample from $f(x; ¯) = \begin{cases} ¯ \; e^{-¯ x} & x > 0, ¯ > 0 \\ o.w. & \end{cases}$

    Obtain the moment estimator for ¯ .
17. Show that the posterior mean is the Bayes estimator with respect to squared error loss.
18. Obtain 100(1 – α)% confidence interval for the difference of means of two normal populations with common unknown variance.

PART C

Answer any **TWO** questions: **(2X20=40 marks)**

19. a). State and establish Cramer-Rao inequality.

b). Let $X_1, X_2, ..., X_n$ be a random sample from Poisson distribution with parameter ª , obtain Cramer Rao lower bound for the variance of unbiased estimators of ª . **(12+8)**

20. a). State and prove Rao –Blackwell theorem.

b). Let X_1, X_2,.., X_n be i.i.d. B(1, θ), 0 < θ < 1. Starting with the UBE X_1, show that we can get a better UBE for θ by conditioning on a sufficient statistic. **(10+10)**

21. a). Give an example to show that Maximum Likelihood Estimator need not be unique.

b). Find the maximum likelihood estimator for simultaneous estimation of « and ±^{2} based on a random sample from N(«, ±^{2}) **(8+12)**

22. a). Let X be B(n, θ), 0 < θ< 1. Obtain Bayes estimator for ¯ by taking uniform prior.

b). Obtain 100 (1 – ~)% asymptotic confidence interval for the parameter p of the Bernoulli distribution. **(12+8)**

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