LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **STATISTICS & MATHEMATICS**

THIRDSEMESTER – NOVEMBER 2017

ST 3876- MATHEMATICAL FINANCE MODELS

Date: 15-11-2017 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

I.(a) If you borrow Rs. 1,000 for one year at an interest rate of 8% per year compounded quarterly, how much do you owe at the end of the year? (5)

(OR)

(b) Explain the risk neutral probability. (5)

(c) Suppose you have just spoken to a bank about borrowing Rs. 1,00,000 to purchase a land and the loan officer has told you that a Rs. 1,00,000 loan, to be repaid in monthly installments over 15 years with an interest rate of 0.6% per month, could be arranged. If the bank charges a loan initiation fee of Rs. 600, a land inspection fee of Rs. 400, and 1 point for the processing fee. What is the effective annual interest rate of the loan being offered? (15)

(OR)

(d)Explain Geometric Brownian motion as a limit simpler model. (15)

2. (a) Find the yield curve and the present value function if $r(s) = \frac{1}{1+s}r_1 + \frac{s}{1+s}r_2$. (5)

(OR)

(b) Prove that the No arbitrage option cost C is decreasing in Strike price K and convex function of s. (5)

(c) State and prove law of one price. Illustrate by an example. (15)

(OR)

(d) Derive the Black Scholes no arbitrage option cost Formula. (15)

3. (a) Prove that the dividend for each share of the security is paid continuously in time at a rate equal to a fixed fraction f of the price of the security. (5)

(OR)

(b)Suppose that a security is presently selling for a price of 60, the nominal rate is 9% (with the unit of time being one year) and the security's volatility is 0.35. Find the no arbitrage cost of a call option that expires in three months and has a strike price 68. (5)

(c) Explain the Delta Hedging Arbitrage Strategy (15)

(OR)

(d) Assuming a Log Normal Distribution for the size of a jump, prove that ,

$$\text{No – arbitrage cost} = \sum_{n=0}^{\infty} e^{-\lambda t E[J]} \frac{(\lambda t E[J])^n}{n!} C(s,t,K,\sigma(n),r(n)) \quad (15)$$

4. (a) Suppose that the current risk-free interest rate is 6% and the expected value and the standard deviation of the market rate of return are 0.10 and 0.20 respectively. If the covariance of the rate of return of a given stock and the market's rate of return is 0.05.

What is the expected rate of return of that stock? (5)

(OR)

(b) Explain in detail, the Expected Value at Risk. (5)

© Suppose that three investment projects with the following return functions are available:

$f_1(x) = \frac{10x}{1+x}, x = 0,1, \ldots$

$f_2(x) = \sqrt{x}\,, x = 0,1, \ldots$

$f_3(x) = 10(1 - e^{-x}), x = 0,1 \ldots$

Find the maximal sum of returns from investing 5. (15)

(OR)

(d) Estimate the volatility parameter when the collection prices follow Geometric Brownian motion using Opening, Closing and High –Low data. (15)

5. (a) Explain barrier call option with a specified strike price. (5)

(OR)

(b)Explain the Gambling model with Unknown Win Probabilities. (5)

(c) Derive the pricing Exotic options by simulation. (15)

(OR)

(d)Derive the pricing Exotic options by simulation. (15)

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