LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIFTH SEMESTER – **NOVEMBER 2017**

ST 5504 – ESTIMATION THEORY

Date: 07-11-2017 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART A

Answer **ALL** the questions: **(10X2=20 marks)**

1. Define consistency of an estimator.
2. Give an example of an estimator which is consistent but not unbiased.
3. Define sufficiency of an estimator.
4. State factorization theorem.
5. Write any two properties of MLE.
6. Describe method of moments for estimating the parameter.
7. What is Baye’s estimator?
8. Define posterior distribution.
9. What do you mean by pivotal quantity?
10. Write 100 $(1-\sim)\%$ confidence interval for « , if a random sample of size n is drawn from $N(«,4)$ « $\in \Re$.

PART B

Answer any **FIVE** questions: **(5X8=40 marks)**

11. State and prove the invariance property of consistent estimator.
12. Prove that UMVUE is unique.
13. State and prove Rao-Blackwell theorem.
14. Describe about method of least squares in estimating the parameters.
15. Let $X_1, X_2, ..., X_n$ be a random sample of size n from $N(10, \pm^2); \pm > 0$. Obtain 100 $(1-\sim)\%$ confidence interval for $\pm^2$.
16. Obtain the MLE for the parameter ¯ , if a random sample of size n is drawn from $U(0, ¯); ¯ \in \Re$.
17. Let $X_1, X_2, ..., X_n$ be a random sample of size n drawn from a distribution

$$f(x; ¯) = \begin{cases} ¯\ x^{¯-1}; & 0 < x < 1, ¯ > 0 \\ 0; & otherwise \end{cases}$$

Obtain sufficient statistic for ¯ .

18. Describe Bayes estimation.

PART C

Answer any **TWO** questions: **(2X20=40 marks)**

19. (i). State and prove Chappman-Robbin's inequality. **(10)**

    (ii). Obtain MVB estimator for μ , if a random sample of size n is drawn from

    $N(\mu, \sigma^2)$ $\mu \in \Re;\ \sigma > 0$ *and known.* **(10)**

20. (i). State and prove Lehmann-Scheffe theorem. **(2+8)**

    (ii). Let $X_1, X_2, ..., X_n$ be a random sample from Bernoulli distribution

    $$f(x;\theta) = \begin{cases} \theta^x (1-\theta)^{1-x}, & x = 0, 1,\ 0 < \theta < 1 \\ 0, & otherwise \end{cases}$$

    Show that $\sum_{i=1}^{n} X_i$ is complete sufficient for θ . **(10)**

21. (i). Let $X_1, X_2, ..., X_n$ be a random sample from $f(x;\theta) = \begin{cases} \theta e^{-\theta x}, & x > 0, \theta > 0 \\ 0, & otherwise \end{cases}$

    Obtain MLE for θ . **(10)**

    (ii). Construct $100\ (1-\alpha)\%$ confidence interval for difference between proportions of two independent binomial populations. **(10)**

22. Let $X \sim b(n,\ p)$ and $L(p, \delta(x)) = (p - \delta(x))^2$. Let $\pi(p) = 1$ for $o < p < 1$ be the prior PDF of p . Find Bayes estimator. **(20)**

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