LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **NOVEMBER 2019**

17/18PST3MC02/ST 3816 – STOCHASTIC PROCESSES

Date: 31-10-2019 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

Section A

Answer all the questions 10 X 2 = 20 marks

1. Define state space and index parameter of a stochastic process.
2. Define communication and periodicity of a Markov chain.
3. Provide any two properties of the period of a state.
4. Write the postulates of birth and death process.
5. Highlight any two applications of renewal process.
6. Define supermartingale.
7. Cite any two examples for stationary process.
8. Prove that recurrence is a class property.
9. Define finite state continuous time Markov chain.
10. Write a note on discrete time branching process.

Section B

Answer any five questions 5X 8 = 40 marks

11. Explain : (i) Spatially homogeneous Markov chains (ii) one-dimensional random walks.
12. Prove that the two-dimensional random walk is recurrent.
13. Discuss the limiting behavior of P_{ij}^{n} when i is transient and j is recurrent.
14. Derive the differential equations for pure birth process.
15. Explain the following functionals of the Poisson process:

    (a)The renewal function (b) Excess life (c) current life (d) Mean total life.
16. Explain Wald′ s martingale.
17. Explain two-type branching process.
18. Show that the moving average process is covariance stationary.

Section C

Answer any two questions 2 X 20 = 40 marks

19.(a) State and prove the basic limit theorem of Markov chains.

(b) Investigate the existence of a stationary probability distribution for the class of random walks whose transition matrices are given by

$$P = \begin{Vmatrix} 0 & 1 & 0 & \ldots\ldots & \\ q_1 & 0 & p_1 & \ldots\ldots.. & \\ 0 & q_2 & 0 & p_2 & \ldots.. \\ . & . & . & . & \\ . & . & . & . & \\ . & . & . & . & \end{Vmatrix}$$

(10 + 10) marks.

20.(a) Derive the backward Kolmogorov differential equations for birth and death process.

(b) Derive M(t) = E [X(t)} for a birth and death process having linear growth with immigration.

(8+12)marks

21. (a) Establish the generating function relations for branching process.

(b) If π is the probability of eventual extinction , show that it is the smallest positive root of the equation $\varphi(s) = s$. (8 + 12) marks.

22. If a Markov chain on states {0, 1, 2, 3, 4, 5} has transition probability matrix

$$P = \begin{Vmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{5} & \frac{4}{5} & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{Vmatrix}$$

(i) Find all classes and check for recurrence of states.

(ii) Compute $\lim_{n\to\infty} p_{5i}^{n}$ for i=0,1,2,3,4,5

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