LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2019**

PST 1501 – ADVANCED DISTRIBUTION THEORY

Date: 30-10-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

SECTION – A

Answer ALL the questions **(10 x 2 = 20 Marks)**

1. Write the pdf of truncated Binomial distribution truncated at 0 and 1.
2. Find $E(X^2)$ for lognormal distribution.
3. Write the MGF of Multinomial distribution. What is the distribution of X_1?
4. Let X_1 and X_2 be independent such that $X_1 \sim N(«_1, ±_1^2)$ and $X_2 \sim N(«_2, ±_2^2)$. Obtain the distribution of $2X_1+3X_2$.
5. Write the pdf of k^{th} order statistics when a random sample of size n is from exponential distribution.
6. Prove the additive property of chi-square distribution.
7. Show that the eigen values of the Idempotent matrix is either 0 or 1.
8. Explain spectral decomposition of a real symmetric matrix.
9. Define Non-central F – distribution.
10. State any 4 properties of distribution function.

SECTION – B

Answer any FIVE questions **(5 x 8 = 40 Marks)**

11. Decompose the following distribution function into discrete and continuous parts. Obtain the mean and variance.

$$F(x) = \begin{cases} 0 & x < -1 \\ \dfrac{x+2}{4} & -1 \le x < 1 \\ 1 & 1 \le x < \infty \end{cases}$$

12. Derive the PGF of Bivariate Binomial distribution. Hence prove the additive property of Bivariate Binomial distribution.
13. Derive the mean and variance of truncated Poisson distribution truncated at 0.

14. Derive the joint pdf of the i^{th} and j^{it} order statistics when a random sample of size n is drawn from a continuous distribution.

15. If X has $N_n(0,\Sigma)$ where Σ is positive definite. Let $Q = X'AX$ for a symmetric matrix A with rank r. prove Q has chi-square with r degrees of freedom iff $A\Sigma A = A$.

16. Obtain the marginal and conditional distributions in the case of Bivariate Normal distribution.

17. Let X_1 and X_2 be iid N(0,1) variables. Obtain the distribution of $\frac{X_1}{X_2}$.

18. Explain the compound distributions. Let X have a Poisson distribution with parameter ^{a} where ^{a} is a continuous random variable with Gamma $(\sim, g)$. Obtain the compound distribution of X.

SECTION – C

Answer any TWO questions **(2 x 20 = 40 Marks)**

19. a) State and Prove Skitovitch theorem.

    b) Let X_1, X_2 be iid continuous type random variables with finite second moment. Show that X_1 is normal iff $X_1 + X_2$ and $X_1 - X_2$ are independent. **(10+10)**

20. State and prove any three characterization properties of Geometric distribution.

21. a) Derive the pdf of non-central chi-square distribution.

    b) Let X_1 and X_2 be independent gamma random variables such that X_1 with Gamma $(\sim_1, m)$ and X_2 with Gamma $(\sim_2, n)$. Obtain the distribution of $\frac{X_1}{X_1 + X_2}$ and $X_1 + X_2$.

    c) State and prove the additive property of Gamma distribution. **(10+6+4)**

22. a) Obtain the PGF of Bivariate Poisson distribution.

    b) Obtain the conditional pgf of X_1 given $X_2=x_2$. Hence obtain the correlation coefficient.

    c) Derive the necessary and sufficient condition for X_1 and X_2 to be independent in a Bivariate Poisson distribution. **(5+10+5)**

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