LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2022**

PST1MC04 – SAMPLING THEORY

Date: 30-11-2022 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION - A

Answer ALL the Questions

1	**Define the following:** (5x1 = 5)		
a)	Probability Sampling Design.	K1	CO1
b)	First and second order inclusion probabilities.	K1	CO1
c)	Unbiased estimator for population total under PPSWR.	K1	CO1
d)	Estimator for population total under Random Group Method.	K1	CO1
e)	Multistage Sampling.	K1	CO1
2	**Fill in the blanks.** (5x1 = 5)		
a)	Property of unbiasedness of a statistic is __________________.	K2	CO1
b)	The mean of the inclusion indicator $I_i(s)$ is ________________.	K2	CO1
c)	The Hurwitz – Thompson Estimator for population total is defined, provided ______.	K2	CO1
d)	Cumulative Total Method is a ____________________selection method.	K2	CO1
e)	$\hat{Y}_{LSS}$ is more efficient than $\hat{Y}_{SRS}$, when the population is __________________.	K2	CO1

SECTION - B

	Answer any THREE of the following Questions. (3x10 = 30)		
3	Suppose from a sample of n units selected using SRS, a subsample of n' units is selected using SRS and included in the original sample. Obtain the expected value and the approximate sampling variance of $\hat{\bar{Y}}'$, the sample mean based on (n + n') units.	K3	CO2
4	Obtain the expression for Π_i and Π_{ij}, under MSD.	K3	CO2
5	Describe Regression Estimation and derive an approximate expression for the bias and MSE of $\hat{Y}_{LR}$	K3	CO2
6	Verify if the Hansen-Hurwitz estimator $\hat{Y}_{dhh}$ under double sampling is unbiased for Y and find $V(\hat{Y}_{dhh})$.	K3	CO2
7	In Two – Stage Sampling with SRS in both stages, obtain the mean and variance of the estimator $\hat{Y}_{TS}$, for estimating population total.	K3	CO2

SECTION - C

Answer any TWO of the following Questions. (2 x 12.5 = 25)

8	Deduce the formula for $\hat{Y}_{St}$, $V(\hat{Y}_{St})$ and $v(\hat{Y}_{St})$ when samples are drawn independently from different strata using (i) SRSWOR and (ii) PPSWR.	K4	CO3

9	A SRS of size n = $n_1 + n_2$ with mean $\hat{Y}$ is drawn from a finite population of N units and a SR subsample of size n_1 is drawn from it with mean $\hat{Y}_1$. Derive $V_P(\hat{Y}_1 - \hat{Y}_2)$, where $\hat{Y}_2$ is the mean of the remaining n_2 units in the sample.	K4	CO3
10	For any design, obtain V ($\hat{Y}_{HT}$).	K4	CO3
11	Check if v ($\hat{Y}_{\mathrm{HT}}$) ≥ 0 for all 's' receiving positive probabilities under MSD.	K4	CO3

SECTION - D

Answer any ONE of the following Questions. **(1 x 15 = 15)**

12	In LSS, under linear population, obtain Yates' corrected estimator for estimating population total without error.	K5	CO4
13	Explain proportional allocation in Stratified Sampling and deduce V ($\hat{Y}_{St}$) under this allocation.	K5	CO4

SECTION - E

Answer any ONE of the following questions. **(1 x 20 = 20)**

14	(a) Derive the approximate expression for B($\hat{Y}_R$) and MSE($\hat{Y}_R$). **(10)** (b) Describe in detail Warner's randomized response method for estimating population proportion $\prod_A$. **(10)**	K6	CO5
15	After the decision to take a SRS has been made, it was realized that Y_1, the value of population unit 1 would be unusually low and Y_N, the value of population unit N would be unusually high. In such cases, it is decided to use the estimator $\hat{Y}^* = \hat{Y} + c$ if $1 \in s,\ N \notin s$, $\hat{Y}^* = \hat{Y} - c$ if $1 \notin s,\ N \in s$, and $\hat{Y}^* = \hat{Y}$ otherwise where 'c' is a predetermined constant. Show that $\hat{Y}^*$ is unbiased for $\bar{Y}$ for any 'c'. Derive V ($\hat{Y}^*$). Find the value of 'c' for which $\hat{Y}^*$ is more efficient than $\hat{Y}$.	K6	CO5

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