LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FOURTH SEMESTER – **NOVEMBER 2022**

UST 4501 – ESTIMATION THEORY

Date: 26-11-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

SECTION – A

Answer ALL the Questions **10 × 2 =20**

1. What is the difference between Parameter and Statistic?
2. Write the four properties of an estimator.
3. Define Sufficient Statistic.
4. State the use of Lehmann-Scheffe Theorem in Estimation Theory.
5. Suggest an Moment estimator of Poisson Distribution with the parameter λ.
6. Describe the Method of Modified Minimum Chi-square estimation.
7. State any two properties of UMVUE.
8. State the 95% confidence interval for μ , when a random sample of size 'n' is drawn from N(μ ,1).
9. Define loss function.
10. What do you understand by Prior Distribution?

SECTION – B

Answer Any FIVE from the following **5 × 8 = 40**

11. State and prove a sufficient condition for an estimator to be consistent.
12. Derive a sufficient statistic of λ in a Poisson distribution, based on a random sample of size 'n'.
13. Derive the Cramer-Rao Lower Bound for estimating μ in N(μ, 1), and obtain minimum variance bound unbiased estimator for μ .
14. Let $X_1, X_2, \ldots, X_n$ be a random sample from a Bernoulli distribution:

$$f(x,\theta) = \begin{cases} \theta^x(1-\theta)^{1-x} & ; x = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

Show that $\sum_{i=1}^{n} X_i$ is a complete sufficient statistics for θ.

15. Give an example to show that Maximum Likelihood Estimator need not be unique.
16. Find the moment estimator for the estimation of μ and σ^2 based on a random sample from N (μ, σ^2).
17. Show that the posterior mean is the Bayes estimator with respect to squared error loss.
18. Obtain 100(1 – α)% confidence interval for the difference of means of two normal populations with common unknown variance.

SECTION – C

Answer any TWO from the following **20 × 2 = 40**

19. a) Show that the family of Poisson distribution {P(λ), λ> 0) } is complete.

b) State and Prove Cramer-Rao Inequality. **(10+10)**

20. a) State and prove Rao –Blackwell theorem. **(10+10)**

b) Derive the MLE of the parameter θ based on a random sample from U[θ – 1, θ + 1].

21. a) State and prove Lehmann-Scheffe theorem.

b) State and prove factorization theorem. **(10+10)**

22. a) Obtain 100 (1−α)% asymptotic confidence interval for the parameter p of the Bernoulli distribution.

b) Let X_1 , X_2 …, X_n be a random sample from Bernoulli distribution B(1,θ). Obtain the Bayes estimator for θ by taking a suitable prior. **(10+10)**

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