LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **NOVEMBER 2023**

PST1MC04 – SAMPLING THEORY

Date: 08-11-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

	SECTION A – K1 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
1	**Define the following**
a)	Inclusion Indicators and Inclusion Probabilities.
b)	Fixed and Varying size sampling design.
c)	Cumulative Total Method.
d)	Ratio Estimator.
e)	Lahiri's Method.
	SECTION A – K2 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
2	**Fill in the blanks**
a)	Formula for MSE of an estimator T in terms of Variance and Bias is__________.
b)	For any Sampling Design P(.), $E_p[n(s)] =$ __________.
c)	An unbiased estimator of Y under random group method is _____________
d)	The ratio estimator is a particular case of ________________________.
e)	In Linear Systematic Scheme, the constant **k** is known as __________________.
	SECTION B – K3 (CO2)
	Answer any THREE of the following **(3 x 10 = 30)**
3	Prove that unbiasedness of an estimator depends on the sampling design.
4	In PPSWOR sampling scheme, give the reason for using Desraj ordered estimator instead of Hurwitz – Thompson estimator. Verify if Desraj ordered estimator is unbiased for population total.
5	a) Show that the estimator $\hat{Y}_{HT}$ is unbiased for Y. b) Verify if $\widehat{Y_{HT}}$ is unbiased for 'Y' using (i) the definition of expectation and (ii) an expression involving inclusion indicators under the design $P(s) = \begin{cases} \frac{1}{7} \; if\ s = \{1,2\} \\ \frac{3}{7} \; if\ s = \{2,3,4\} \\ \frac{3}{7} \; if\ s = \{3,4,5\} \\ 0, otherwise \end{cases}$ Given $Y_1 = 4$, $Y_2 = 3$, $Y_3 = 5$, $Y_4 = 2$, and $Y_5 = 7$.
6	Explain Random Group Method in detail. Also, prove that an unbiased estimator of Y under random group method is $\hat{Y}_{RG} = \sum_{i=1}^{n} \frac{y_i}{x_i} T_x(i)$.

7	Explain proportional allocation in Stratified Sampling and deduce $V(\hat{Y}_{St})$ under this allocation.
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8	Examine whether $T_1(s) = \frac{1}{n(s)} \sum_{i \in s} Y_i$ and $T_2(s) = [\max_{i \in s}\{Y_i\} + \min_{i \in s}\{Y_i\}] / 2$ are unbiased for $\bar{Y}$ under the sampling design $P(s) = \begin{cases} \frac{1}{4} & if\ n(s) = 3 \\ 0 & otherwise \end{cases}$. Given $Y_1 = 7$, $Y_2 = 3$, $Y_3 = 4$, and $Y_4 = 2$.
9	If the UBE of $V(\hat{Y}_{HT})$ is $v(\hat{Y}_{HT}) = \sum_{i \in s} \sum_{i \in s} \left(\frac{Y_i}{\pi_i} - \frac{Y_j}{\pi_j}\right)^2 \left(\frac{\pi_i \pi_j - \pi_{ij}}{\pi_{ij}}\right); i < j$, then under SRSWOR, show that $v(\hat{Y}_{HT}) = \frac{N(N-n)}{n} s_y^2$.
10	Obtain Hartley – Ross unbiased ratio type estimator for population total.
11	Explain in detail Warner's Model and find the estimated variance of $\widehat{\Pi}_A$
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 15 = 15)**
12	Under Midzuno sampling Design, show that (i) the first order inclusion probability is $\Pi_i = \frac{N-n}{N-1} . \frac{X_i}{X} + \frac{n-1}{N-1}$, i=1,2…, N, and (7) (ii) the second order inclusion probability is $\Pi_{ij} = \frac{(N-n)(n-1)}{(N-1)(N-2)} . \frac{X_i + X_j}{X} + \frac{(n-1)(n-2)}{(N-1)(N-2)}$, i ≠ j = 1, 2, …, N. (8)
13	a. Obtain the approximate bias and MSE of the Ratio Estimator. (7) b. Verify if the Hansen-Hurwitz estimator $\hat{Y}_{dhh}$ under double sampling is unbiased for Y and find V($\hat{Y}_{dhh}$). (8)
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
14	a. A Simple Random Sample of size $n = n_1 + n_2$ with mean $\hat{\bar{Y}}$ is drawn from a finite population of N units and a Simple Random Subsample of size n_1 is drawn from it with mean $\hat{\bar{Y}}_1$. Show that $Var_p\left(\hat{\bar{Y}}_1 - \hat{\bar{Y}}_2\right) = S_y^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)$, where $\hat{\bar{Y}}_2$ is the mean of the remaining n_2 units in the sample. (12) b. Check if $v(\hat{Y}_{HT})$ is non-negative under MSD for all 's' receiving positive probabilities. (8)
15	**a.** Describe Simmon's Unrelated Randomized Response Model and estimate the population proportion Π_A when Π_Y is unknown. (12) **b.** Describe the linear regression estimation procedure for estimating the population total "Y". (8)

&&&&&&&&&&