LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – STATISTICS

THIRD SEMESTER – NOVEMBER 2023

PST3MC02 – ADVANCED STOCHASTIC PROCESSES

Date: 01-11-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

	SECTION A – K1 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
1	Define the following
a)	Markov process
b)	Periodicity of Markov chain
c)	Renewal process
d)	Super- martingale
e)	Reflected Brownian motion
	SECTION A – K2 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
2	Fill in the blanks
a)	A matrix is called Markov if each row sum is ----------.
b)	Recurrence is a ---------- property.
c)	For Poisson process the inter-occurrence times is ------------------ distribution.
d)	In counter models the readjustment period is called ---------- time.
e)	If $\varphi(s) = p_0 + p_{1.}\ s$, $0 < p_0 < 1$, the associated branching process is called a pure ------- process.
	SECTION B – K3 (CO2)
	Answer any THREE of the following **(3 x 10 = 30)**
3	Explain spatially homogeneous Markov chains.
4	Show that one-dimensional random walk is recurrent.
5	State the postulates for a pure birth process and derive the differential equations for it.
6	Narrate the branching process with two examples.
7	Discuss age and block replacement policies.
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8	Analyze Type I and Type II counter models in renewal process.
9	Obtain mean and variance of Yule process when X(0) = N = 1.
10	Establish the following: (i)The variance of sum as a martingale and (ii)Wald's martingale. (6.5+6)
11	Elaborate the stationary process considering certain trigonometric polynomials
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 15 = 15)**
12	If π denotes the probability of eventual extinction show that it is the smallest positive root of the equation $\varphi(s) = s$ and also prove that $\pi = 1$ if $m \leq 1$ and $0 < \pi < 1$ if $m > 1$.
13	Let a Markov chain on the states $\{0,1,2,3,4,5\}$ has the following one-step transition probabilities: $P_{00} = 1$, $P_{11} = 3/4$, $P_{12} = 1/4$, $P_{21} = 1/8$, $P_{22} = 7/8$, $P_{30} = P_{31} = 1/4$, $P_{33} = 1/8$, $P_{34} = 3/8$, $P_{40} = 1/3$, $P_{42} = P_{43} = 1/6$, $P_{44} = 1/3$, $P_{55} = 1$. (a)Find the equivalence classes. (b)Find period for different classes. (c) Find out the recurrent and transient states. (3+3+9)

SECTION E – K6 (CO5)	
	Answer any ONE of the following **(1 x 20 = 20)**
14	(a)Show that Poisson process can be viewed as a renewal process and (b) State and prove the elementary renewal theorem. (10+10)
15	Establish the probability generating function relations for branching process and obtain mean and variance for it.

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